4-5 Counting Principles

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5. *[Maximum mark: 5]*

(a) Show that $\frac{\sin 2\theta}{1+\cos 2\theta} = \tan\theta$. *[2 marks]*

(b) Hence find the value of $\cot\frac{\pi}{8}$ in the form $a+b\sqrt{2}$, where $a, b \in \mathbb{Z}$. *[3 marks]*

a.) $\frac{\sin 2\theta}{1+\cos(2\theta)} = \tan\theta$

$\frac{2\sin\theta\cos\theta}{\cancel{1}+2\cos^2\theta-\cancel{1}} =$

$\frac{\cancel{2}\sin\theta\cancel{\cos\theta}}{\cancel{2}\cos^{\cancel{2}}\theta} =$

$\frac{\sin\theta}{\cos} =$

$\tan\theta = \tan\theta$

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b.) $\cot\theta = \dfrac{1+\cos(2\theta)}{\sin(2\theta)}$

$\cot\frac{\pi}{8} = \dfrac{1+\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}}$

$= \dfrac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = \dfrac{2}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} + 1$

$= \dfrac{\cancel{2}\sqrt{2}}{\cancel{2}} + 1 = \sqrt{2}+1$

$\boxed{\begin{array}{l} a=1 \\ b=1 \end{array}}$

Opener

a.) Nine poker chips each carrying the numerials 1-9 ar
placed in a box. Two chips are chosen such that the firs
chip is chosen, the number is recorded, the chip is replaced, and then the second chip is chosen. The numbers on the chips are added. How many different ways can you get a sum of 8?

1,7 7,1
2,6 6,2
3,5 5,3
4,4

7 ways

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b.) Suppose now that the first chip is chosen, the number is recorded, and chip is not put back in box before the second chip is chosen. In how many ways can you get a sum of 8?

1, 7   7, 1
2, 6   6, 2
3, 5   5, 3
~~4, 4~~

6 ways

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The difference between the two situations is described by saying that the first random selection is done **with replacement** while the second is done **without replacement**.

Fundamental Principle of Counting

If there are ***m*** ways an event can occur and ***n*** ways a second event can occur, then there are ***m*** x ***n*** ways that the two events can occur. This can be extended to more than two events.

Ex1. Diane is ordering pizza for her family. There are 4 different possible sizes of the pizza. Also, she has to choose 1 of 5 toppings to place on the pizza and 1 of 3 different types of cheese. In addition, she must choose 1 of 3 different kinds of crust. How many different ways can she order her pizza?

$4 \cdot 5 \cdot 3 \cdot 3 = 120$ ways

$\underline{3} \cdot \underline{2} \cdot \underline{1} = 6$

Ex2

a.) You go to a restaurant and order a fruit salad. They have 5 different kinds of fruit and you get to choose 3 to put on your salad. How many different choices do you have?

$\underline{5} \cdot \underline{4} \cdot \underline{3} = 60$

$\frac{60}{6} = 10$ ways

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Ex2

b.) A particular bank has 3 digit PIN numbers where a digit cannot be repeated. How many PINS are available.

$\underline{10} \cdot \underline{9} \cdot \underline{8} = 720$ ways

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If order doesn't matter, it is a **combination**.

If order matters, it is a **permutation**.

Is a combination lock a combination or permutation?

The number of permutations of n objects taken r at a time (without replacement)

The number of permutations of n objects taken r at a time is:

$$nPr = \frac{n!}{(n-r)!}$$

The number of combinations of n objects taken r at a time (without replacement)

The number of combinations of n objects taken r at a time is:

$$nCr = \frac{n!}{(n-r)!r!}$$

Ex3.

order doesn't matter
combination

a.) How many different teams of 5 can be selected from a squad of 12.

$$\frac{12!}{7!5!} = 792$$

Ex3.

c.) a committee of 5 is chosen from 10 men and 6 women. Determine the number of ways of selecting the committee if:

i.) there are no restrictions

ii.) it must contain 3 men and 2 women

iii.) it must contain all women

iv.) It must contain at least 3 men

v.) It must contain at least 1 of each sex.

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